

Section 4.4

Quiz Solutions Spring 2020

① #28 (4.4)

$$\begin{aligned} & \lim_{x \rightarrow 0} \frac{\sinh x - x}{x^3} \quad \text{form } \begin{pmatrix} 0 \\ 0 \end{pmatrix} \\ \stackrel{L}{=} & \lim_{x \rightarrow 0} \frac{\cosh x - 1}{3x^2} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \\ \stackrel{L}{=} & \lim_{x \rightarrow 0} \frac{\sinh x}{6x} \begin{pmatrix} 0 \\ 0 \end{pmatrix} \stackrel{L}{=} \lim_{x \rightarrow 0} \frac{\cosh x}{6} = \boxed{\frac{1}{6}} \end{aligned}$$

② #53 (4.4)

$$\lim_{x \rightarrow 0^+} \left(\frac{1}{x} - \frac{1}{e^x - 1} \right) \quad \infty - \infty$$

$$\lim_{x \rightarrow 0^+} \left(\frac{e^x - 1 - x}{x(e^x - 1)} \right) \quad \frac{0}{0}$$

$$\stackrel{L}{=} \lim_{x \rightarrow 0^+} \frac{e^x - 1}{xe^x + e^x - 1}$$

$$\stackrel{L}{=} \lim_{x \rightarrow 0^+} \frac{e^x}{xe^x + e^x + e^x} = \frac{1}{0+1+1} = \boxed{\frac{1}{2}}$$

$$\textcircled{3} \quad \#63 \quad \lim_{x \rightarrow \infty} x^{\frac{1}{x}} \quad \textcircled{\infty^0}$$

$$y = x^{\frac{1}{x}}$$

$$\ln y = \ln x^{\frac{1}{x}}$$

$$\ln y = \frac{1}{x} \cdot \ln x$$

$$\ln y = \lim_{x \rightarrow \infty} \frac{1}{x} \ln x \quad \left(\frac{\infty}{\infty} \right)$$

$$\ln y = \lim_{x \rightarrow \infty} \frac{\frac{1}{x}}{1} = \frac{1}{\infty} \rightarrow 0$$

$$\ln y = 0$$

$$y = e^0 = \textcircled{1}$$